0.1 `threesls`: Three Stage Least Squares

`threesls` is a combination of two stage least squares and seemingly unrelated regression. It provides consistent estimates for linear regression models with explanatory variables correlated with the error term. It also extends ordinary least squares analysis to estimate system of linear equations with correlated error terms

Syntax

```
> fml <- list(mu1 = Y1 ~ X1 + Z1, mu2 = Y2 ~ X2 + Z2, inst1 = Z1 ~
+      W1 + X1, inst2 = Z2 ~ W2 + X2)
```

```
> z.out <- zelig(formula = fml, model = "treesls", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)
```

Inputs

`threesls` regression specification requires at least two sets of equations. The first set of M euqations corresponds to the M dependent variables $(Y_1, \ldots, Y_M)$ to be estimated. The second set of equations (Z) corresponds to the instrumental variables in the M equations.

- `formula`:a list of the system of equations and instrumental variable equations. The system of equations is listed first as `mus`. The equations for the instrumental variables are listed next as `insts`. For example:

  ```
  > fml <- list(mu1 = Y1 ~ X1 + Z1, mu2 = Y2 ~ X2 + Z2, inst1 = Z1 ~
  +      W1 + X1, inst2 = Z2 ~ W2 + X2)
  ```

  `"mu1"` is the first equation in the two equation model with `Y1` as the dependent variable and `X1` and `Z1` as the explanatory variables. `"mu2"` is the second equation with `Y2` as the dependent variable and `X2` and `Z2` as the explanatory variables. `Z1` and `Z2` are also problematic endogenous variables, so they are estimated through instruments in the `"inst1"` and `"inst2"` equations.

- `Y`: dependent variables of interest in the system of equations.

- `Z`: the problematic explanatory variables correlated with the error term.

- `W`: exogenous instrument variables used to estimate the problematic explanatory variables (`Z`)

Additional Inputs

threesls takes the following additional inputs for model specifications:

- TX: an optional matrix to transform the regressor matrix and, hence, also the coefficient vector (see details). Default is NULL.
- maxiter: maximum number of iterations.
- tol: tolerance level indicating when to stop the iteration.
- rcovformula: formula to calculate the estimated residual covariance matrix (see details). Default is equal to 1.
- formulathreesls: formula for calculating the threesls estimator, one of "GLS", "IV", "GMM", "Schmidt", or "Eviews" (see details.)
- probdfsys: use the degrees of freedom of the whole system (in place of the degrees of freedom of the single equation to calculate probability values for the t-test of individual parameters.
- single.eq.sigma: use different σ^2 for each single equation to calculate the covariance matrix and the standard errors of the coefficients.
- solvetol: tolerance level for detecting linear dependencies when inverting a matrix or calculating a determinant. Default is solvetol=.Machine$double.eps.
- saveMemory: logical. Save memory by omitting some calculation that are not crucial for the basic estimate (e.g McElroy's R^2).

Details

The matrix TX transforms the regressor matrix (X) by $X* = X \times TX$. Thus, the vector of coefficients is now $b = TX \times b*$ where b is the original(stacked) vector of all coefficients and $b*$ is the new coefficient vector that is estimated instead. Thus, the elements of vector b and $b_i = \sum_j TX_{ij} \times b_j*$. The TX matrix can be used to change the order of the coefficients and also to restrict coefficients (if TX has less columns than it has rows). If iterated (with maxit>1), the covergence criterion is

$$\sqrt{\frac{\sum_i (b_{i,g} - b_{i,g-1})^2}{\sum_i b_{i,g-1}^2}} < tol$$

where $b_{i,g}$ is the ith coefficient of the gth iteration step. The formula (rcovformula to calculate the estimated covariance matrix of the residuals($\hat{\Sigma}$)can be one of the following (see Judge et al., 1955, p.469): if rcovformula$= 0$:

$$\hat{\sigma_{ij}} = \frac{\hat{e_i}\prime\hat{e_j}}{T}$$

if `rcovformula`= 1 or `rcovformula`='geomean':

$$\hat{\sigma_{ij}} = \frac{\hat{e_i}\prime\hat{e_j}}{\sqrt{(T-k_i)\times(T-k_j)}}$$

if `rcovformula`= 2 or `rcovformula`='Theil':

$$\hat{\sigma_{ij}} = \frac{\hat{e_i}\prime\hat{e_j}}{T-k_i-k_j+tr[X_i(X_i\prime X_i)^{-1}X_i\prime X_j(X_j\prime X_j)^{-1}X_j\prime]}$$

if `rcovformula`= 3 or `rcovformula`='max':

$$\hat{\sigma_{ij}} = \frac{\hat{e_i}\prime\hat{e_j}}{T-max(k_i,k_j)}$$

If $i = j$, formula 1, 2, and 3 are equal. All these three formulas yield unbiased estimators for the diagonal elements of the residual covariance matrix. If $ineqj$, only formula 2 yields an unbiased estimator for the residual covariance matrix, but it is not necessarily positive semidefinit. Thus, it is doubtful whether formula 2 is really superior to formula 1 (Theil, 1971, p.322). The formulas to calculate the threesls estimator lead to identical results if the same instruments are used in all equations. If different instruments are used in the different equations, only the GMM-threesls estimator ("GMM") and the threesls estimator proposed by Schmidt (1990) ("Schmidt") are consistent, whereas "GMM" is efficient relative to "Schmidt" (see Schmidt, 1990).

Examples

Attaching the example dataset:

```
> data(kmenta)
```

Formula:

```
> formula <- list(mu1 = q ~ p + d, mu2 = q ~ p + f + a, inst = ~d +
+     f + a)
```

Estimating the model using `threesls`:

```
> z.out <- zelig(formula = formula, model = "threesls", data = kmenta)
> summary(z.out)
```

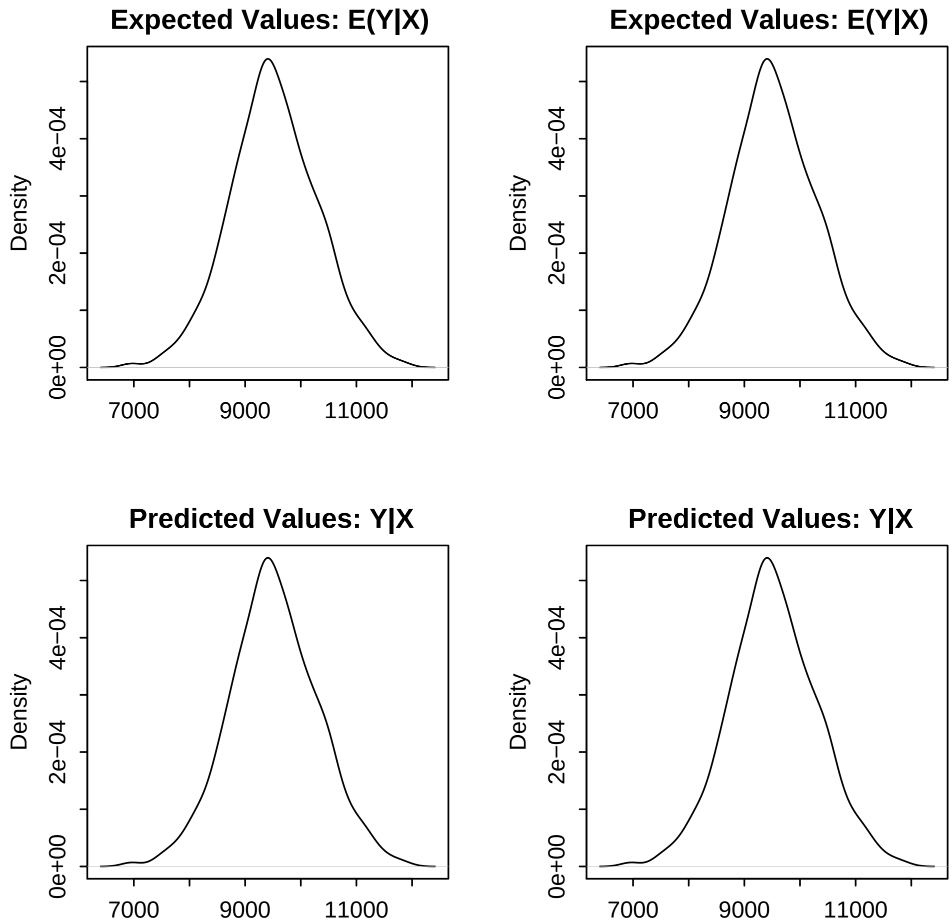
Set explanatory variables to their default (mean/mode) values

```
> x.out <- setx(z.out)
```

Simulate draws from the posterior distribution:

```
> s.out <- sim(z.out, x = x.out)
> summary(s.out)
```

Plot the quantities of interest



Model

See Also

For information about two stage least square regression, see Section **??** and `help(2sls)`. For information about seemingly unrelated regression, see Section **??** and `help(sur)`.

Quantities of Interest

Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run:

```
z.out <- zelig(formula=fml, model = "threesls", data)
```

then you may examine the available information in `z.out` by using `names(z.out)`, see the draws from the posterior distribution of the `coefficients` by using `z.out$coefficients`, and view a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below:

- `rcovest`: residual covariance matrix used for estimation.
- `mcelr2`: McElroys R-squared value for the system.
- `h`: matrix of all (diagonally stacked) instrumental variables.
- `formulathreesls`: formula for calculating the threesls estimator
- `method`: Estimation method.
- `g`: number of equations.
- `n`: total number of observations.
- `k`: total number of coefficients.
- `ki`: total number of linear independent coefficients.
- `df`: degrees of freedom of the whole system.
- `iter`: number of iteration steps.
- `b`: vector of all estimated coefficients.
- `t`: t values for b.
- `se`: estimated standard errors of b.
- `bt`: coefficient vector transformed by TX.

- `p`: p values for b.
- `bcov`: estimated covariance matrix of b.
- `btcov`: covariance matrix of bt.
- `rcov`: estimated residual covariance matrix.
- `drcov`: determinant of `rcov`.
- `rcor`: estimated residual correlation matrix.
- `olsr2`: system OLS R-squared value.
- `y`: vector of all (stacked) endogenous variables.
- `x`: matrix of all (diagonally stacked) regressors.
- `data`: data frame of the whole system (including instruments).
- `TX`: matrix used to transform the regressor matrix.
- `rcovformula`: formula to calculate the estimated residual covariance matrix.
- `probdfsys`: system degrees of freedom to calculate probability values?.
- `solvetol`: tolerance level when inverting a matrix or calculating a determinant.
- `eq`: a list that contains the results that belong to the individual equations.
- `eqnlabel*`: the equation label of the ith equation (from the labels list).
- `formula*`: model formula of the ith equation.
- `n*`: number of observations of the ith equation.
- `k*`: number of coefficients/regressors in the ith equation (including the constant).
- `ki*`: number of linear independent coefficients in the ith equation (including the constant differs from k only if there are restrictions that are not cross equation).
- `df*`: degrees of freedom of the ith equation.
- `b*`: estimated coefficients of the ith equation.
- `se*`: estimated standard errors of b of the ith equation.
- `t*`: t values for b of the ith equation.
- `p*`: p values for b of the ith equation.

- `covb*`: estimated covariance matrix of b of the ith equation.
- `y*`: vector of endogenous variable (response values) of the ith equation.
- `x*`: matrix of regressors (model matrix) of the ith equation.
- `data*`: data frame (including instruments) of the ith equation.
- `fitted*`: vector of fitted values of the ith equation.
- `residuals*`: vector of residuals of the ith equaiton.
- `ssr*`: sum of squared residuals of the ith equation.
- `mse*`: estimated variance of the residuals (mean of squared errors) of the ith equation.
- `s2*`: estimated variance of the residents($\hat{sigma}^2$) of the ith equation.
- `rmse*`: estimated standard error of the reiduals (square root of mse) of the ith equation.
- `s*`: estimated standard error of the residuals ($\hat{\sigma}$) of the ith equation.
- `r2*`: R-squared (coefficient of determination).
- `adjr2*`: adjusted R-squared value.
- `inst*`: instruments of the ith equation.
- `h*`: matrix of instrumental variables of the ith equation.
- `zelig.data`: the input data frame if `save.data = TRUE`.

How to Cite

To cite the *threesls* Zelig model:

> Ferdinand Alimadhi, Ying Lu, and Elena Villalon. 2007. "threesls: Three Stage Least Squares," in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software," `http://gking.harvard.edu/zelig`.

To cite Zelig as a whole, please reference these two sources:

> Kosuke Imai, Gary King, and Olivia Lau. 2007. "Zelig: Everyone's Statistical Software," `http://GKing.harvard.edu/zelig`.
>
> Kosuke Imai, Gary King, and Olivia Lau. 2007. "Toward A Common Framework for Statistical Analysis and Development," `http://gking.harvard.edu/files/abs/z-abs.shtml`.

See also

The threesls function is adapted from the `systemfit` library (Hamann and Henningsen 2005).

Bibliography

Hamann, J. and Henningsen, A. (2005), *systemfit: Simultaneous Equation Systems in R Package*.