

# The Parma Polyhedra Library

## Prolog Language Interface

### User's Manual\*

(version 0.10pre34)

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For the most up-to-date information see the Parma Polyhedra Library site:

<http://www.cs.unipr.it/ppl/>

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## 1 Prolog Language Interface

The Parma Polyhedra Library comes equipped with a Prolog interface. Despite the lack of standardization of Prolog’s foreign language interfaces, the PPL Prolog interface supports several Prolog systems and, to the extent this is possible, provides a uniform view of the library from each such systems.

The system-independent features of the library are described in Section [System-Independent Features](#). Section [Compilation and Installation](#) explains how the various incarnations of the Prolog interface are compiled and installed. Section [System-Dependent Features](#) illustrates the system-dependent features of the interface for all the supported systems.

In the sequel, `prefix` is the prefix under which you have installed the library (typically `/usr` or `/usr/local`).

### System-Independent Features

The Prolog interface provides access to the numerical abstractions (convex polyhedra, BD shapes, octagonal shapes, etc.) implemented by the PPL library. A general introduction to the numerical abstractions, their representation in the PPL and the operations provided by the PPL is given in the main PPL user manual. Here we just describe those aspects that are specific to the Prolog interface.

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  - [MIP Predicates](#),
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## Overview

First, here is a list of notes with general information and advice on the use of the interface.

- The numerical abstract domains available to the Prolog user consist of the *simple* domains, *powersets* of a simple domain and *products* of simple domains.
  - The simple domains are:
    - \* convex polyhedra, which consist of `C_Polyhedron` and `NNC_Polyhedron`;
    - \* weakly relational, which consist of `BD_Shape_N` and `Octagonal_Shape_N` where `N` is one of the numeric types `short`, `signed_char`, `int`, `long`, `long_long`, `mpz_class`, `mpq_class`;
    - \* boxes which consist of `Int8_Box`, `Int16_Box`, `Int32_Box`, `Int64_Box`, `UInt8_Box`, `UInt16_Box`, `UInt32_Box`, `UInt64_Box`, `Double_Box`, `Long_Double_Box`, `Z_Box`, `Rational_Box`, `Float_Box`; and
    - \* the Grid domain.
  - The powerset domains are `Pointset_Powerset_S` where `S` is a simple domain.
  - The product domains consist of `Direct_Product_S_T`, `Smash_Product_S_T` and `Constraints_Product_S_T` where `S` and `T` are simple domains.
- In the following, any of the above numerical abstract domains is called a *PPL domain* and any element of a PPL domain is called a *PPL object*.
- The Prolog interface files are all installed in the directory `prefix/lib/ppl`. Since this includes shared and dynamically loaded libraries, you must make your dynamic linker/loader aware of this fact. If you use a GNU/Linux system, try the commands `man ld.so` and `man ldconfig` for more information.
- The Prolog interface to the PPL is initialized and finalized by the predicates `ppl_initialize/0` and `ppl_finalize/0`. Thus the only interface predicates callable after `ppl_finalize/0` are `ppl_finalize/0` itself (this further call has no effect) and `ppl_initialize/0`, after which the interface's services are usable again. Some Prolog systems allow the specification of initialization and deinitialization functions in their foreign language interfaces. The corresponding incarnations of the PPL-Prolog interface have been written so that `ppl_initialize/0` and/or `ppl_finalize/0` are called automatically. Section [System-Dependent Features](#) will detail in which cases initialization and finalization is automatically performed or is left to the Prolog programmer's responsibility. However, for portable applications, it is best to invoke `ppl_initialize/0` and `ppl_finalize/0` explicitly: since they can be called multiple times without problems, this will result in enhanced portability at a cost that is, by all means, negligible.
- A PPL object such as a polyhedron can only be accessed by means of a Prolog term called a *handle*. Note, however, that the data structure of a handle, is implementation-dependent, system-dependent and version-dependent, and, for this reason, deliberately left unspecified. What we do guarantee is that the handle requires very little memory.

- A Prolog term can be bound to a valid handle for a PPL object by using predicates such as

```
ppl_new_C_Polyhedron_from_space_dimension/3,
ppl_new_C_Polyhedron_from_C_Polyhedron/2,
ppl_new_C_Polyhedron_from_constraints/2,
ppl_new_C_Polyhedron_from_generators/2,
```

These predicates will create or copy a PPL polyhedron and construct a valid handle for referencing it. The last argument is a Prolog term that is unified with a new valid handle for accessing this polyhedron.

- As soon as a PPL object is no longer required, the memory occupied by it should be released using the PPL predicate such as `ppl_delete_Polyhedron/1`. To understand why this is important, consider a Prolog program and a variable that is bound to a Herbrand term. When the variable dies (goes out of scope) or is uninstantiated (on backtracking) the term it is bound to is amenable to garbage collection. But this only applies for the standard domain of the language: Herbrand terms. In Prolog+PPL, when a variable bound to a handle for a PPL Polyhedron dies or is uninstantiated, the handle can be garbage-collected, but the polyhedra to which the handle refers will not be released. Once a handle has been used as an argument in `ppl_delete_Polyhedron/1`, it becomes invalid.
- For a PPL object with space dimension  $k$ , the identifiers used for the PPL variables must lie between 0 and  $k - 1$  and correspond to the indices of the associated Cartesian axes. For example, when using the predicates that combine PPL polyhedra or add constraints or generators to a representation of a PPL polyhedron, the polyhedra referenced and any constraints or generators in the call should follow all the (space) dimension-compatibility rules stated in Section Representations of Convex Polyhedra of the main PPL user manual.
- As explained above, a polyhedron has a fixed topology C or NNC, that is determined at the time of its initialization. All subsequent operations on the polyhedron must respect all the topological compatibility rules stated in Section Representations of Convex Polyhedra of the main PPL user manual.
- Any application using the PPL should make sure that only the intended version(s) of the library are ever used. Predicates

```
ppl_version_major/1,
ppl_version_minor/1,
ppl_version_revision/1,
ppl_version_beta/1,
ppl_version/1,
ppl_banner.
```

allow run-time checking of information about the version being used.

## Predicate Specifications

The PPL predicates provided by the Prolog interface are specified below. The specification uses the following grammar rules:

|            |                       |   |
|------------|-----------------------|---|
| Number     | --> unsigned integer  | ranging from 0 to an upper bound depending on the actual Prolog system. |
| C_int      | --> Number   - Number | C integer   |
| C_unsigned | --> Number            | C unsigned integer  |

|                       |   |   |
|-----------------------|---|---|
| Coeff                 | --> Number  | used in linear expressions;<br>the upper bound will depend on how<br>the PPL has been configured  |
| Dimension_Type        | --> Number  | used for the number of affine and<br>space dimensions and the names of<br>the dimensions;<br>the upper bound will depend on<br>the maximum number of dimensions<br>allowed by the PPL<br>(see <code>ppl_max_space_dimensions/1</code> ) |
| Boolean               | --> true   false  |   |
| Handle                | --> Prolog term   | used to identify a Polyhedron   |
| Topology              | --> c   nnc   | Polyhedral kind;<br>c is closed and nnc is NNC  |
| VarId                 | --> Dimension_Type  | variable identifier   |
| PPL_Var               | --> '\$VAR' (VarId)   | PPL variable  |
| Lin_Expr              | --> PPL_Var<br>  Coeff<br>  Lin_Expr<br>  - Lin_Expr<br>  Lin_Expr + Lin_Expr<br>  Lin_Expr - Lin_Expr<br>  Coeff * Lin_Expr<br>  Lin_Expr * Coeff                                      | PPL variable<br><br>unary plus<br>unary minus<br>addition<br>subtraction<br>multiplication<br>multiplication  |
| Relation_Symbol       | --> =<br>  <=<br>  >=<br>  <<br>  >   | equals<br>less than or equal<br>greater than or equal<br>strictly less than<br>strictly greater than  |
| Constraint            | --> Lin_Expr Relation_Symbol Lin_Expr   | constraint  |
| Constraint_System     | --> []<br>  [Constraint   Constraint_System]  | list of constraints   |
| Generator_Denominator | --> Coeff<br>  Coeff   - Coeff  | must be non-zero  |
| Generator             | --> point(Lin_Expr)<br>  point(Lin_Expr, Generator_Denominator)<br>  closure_point(Lin_Expr)<br>  closure_point(Lin_Expr, Generator_Denominator)<br>  ray(Lin_Expr)<br>  line(Lin_Expr) | point<br>point<br>closure point<br>closure point<br>ray<br>line   |
| Generator_System      | --> []<br>  [Generator   Generator_System]  | list of generators  |
| Atom                  | --> Prolog atom   |   |
| Universe_or_Empty     |   | polyhedron  |

```

--> universe
| empty

Poly_Relation          polyhedron relation:
--> is_disjoint          with a constraint
| strictly_intersects    with a constraint
| is_included            with a constraint
| saturates              with a constraint
| subsumes               with a generator

Relation_List          list of polyhedron relations
--> []
| [Poly_Relation | Relation_List]

Complexity --> polynomial | simplex | any

Rational_Numerator
--> Coeff | - Coeff

Rational_Denominator
--> Coeff                must be non-zero

Rational --> Rational_Numerator rational number
| Rational_Numerator/Rational_Denominator

Bound --> c(Rational)      closed rational limit
| o(Rational)             open rational limit
| o(pinf)                 unbounded in the positive direction
| o(minf)                 unbounded in the negative direction

Interval --> i(Bound, Bound) rational interval

Box --> []
| [Interval | Box]

Vars_Pair --> PPLVar - PPLVar map relation

P_Func --> []
| [Vars_Pair | P_Func].

Optimization_Mode
--> max | min

MIP_Problem_Status
--> unfeasible | unbounded | optimized

Control_Parameter_Name
--> pricing

Control_Parameter_Value
--> pricing_steepest_edge_float
| pricing_steepest_edge_exact
| pricing_rextbook

Vars_List --> []
| [PPL_Var | Vars_List].

```

## Predicate Descriptions

Below is a short description of many of the interface predicates. For full definitions of terminology used here, see the main PPL user manual.

**Domain Independent Predicates** First we describe the domain independent predicates that are included with all instantiations of the Prolog interfaces.

`ppl_version_major(?C_int)`

*Unifies C\_int with the major number of the PPL version.*

`ppl_version_minor(?C_int)`

*Unifies C\_int with the minor number of the PPL version.*

`ppl_version_revision(?C_int)`

*Unifies C\_int with the revision number of the PPL version.*

`ppl_version_beta(?C_int)`

*Unifies C\_int with the beta number of the PPL version.*

`ppl_version(?Atom)`

*Unifies Atom with the PPL version.*

`ppl_banner(?Atom)`

*Unifies Atom with information about the PPL version, the licensing, the lack of any warranty whatsoever, the C++ compiler used to build the library, where to report bugs and where to look for further information.*

`ppl_Coefficient_is_bounded`

*Succeeds if and only if the Coefficients in the C++ interface are bounded.*

`ppl_Coefficient_max(Max)`

*If the Coefficients in the C++ interface are bounded, then the maximum coefficient the C++ interface can handle is unified with Max. If the Prolog system cannot handle this coefficient, then an exception is thrown. It fails if the Coefficients in the C++ interface are unbounded.*

`ppl_Coefficient_min(Min)`

*If the Coefficients in the C++ interface are bounded, then the minimum coefficient the C++ interface can handle is unified with Min. If the Prolog system cannot handle this coefficient, then an exception is thrown. It fails if the Coefficients in the C++ interface are unbounded.*

`ppl_max_space_dimension(?Dimension_Type)`

*Unifies Dimension\_Type with the maximum space dimension this library can handle.*

`ppl_initialize`

*Initializes the PPL interface. Multiple calls to `ppl_initialize` does no harm.*

`ppl_finalize`

*Finalizes the PPL interface. Once this is executed, the next call to an interface predicate must either be to `ppl_initialize` or to `ppl_finalize`. Multiple calls to `ppl_finalize` does no harm.*

`ppl_set_timeout_exception_atom(+Atom)`

*Sets the atom to be thrown by timeout exceptions to Atom. The default value is `time_out`.*

`ppl_timeout_exception_atom(?Atom)`

*The atom to be thrown by timeout exceptions is unified with Atom.*

`ppl_set_timeout(+C_unsigned)`

*Computations taking exponential time will be interrupted some time after C\_unsigned ms after that call. If the computation is interrupted that way, the current timeout exception atom will be thrown. C\_*

*unsigned must be strictly greater than zero.*

`ppl_reset_timeout`

*Resets the timeout time so that the computation is not interrupted.*

`ppl_set_rounding_for_PPL`

*Sets the FPU rounding mode so that the PPL abstractions based on floating point numbers work correctly. This is performed automatically at initialization-time. Calling this function is needed only if `restore_pre_PPL_rounding()` has been previously called.*

`ppl_restore_rounding_for_PPL`

*Sets the FPU rounding mode as it was before initialization of the PPL. After calling this function it is absolutely necessary to call `set_rounding_for_PPL()` before using any PPL abstractions based on floating point numbers. This is performed automatically at finalization-time.*

**MIP Predicates** Here we describe the predicates available for PPL objects defining mixed integer (linear) programming problems.

`ppl_new_MIP_Problem_from_space_dimension(+Dimension_Type, -Handle)`

*Creates an MIP Problem MIP with the feasible region the vector space of dimension `Dimension_Type`, objective function 0 and optimization mode `max`. `Handle` is unified with the handle for MIP.*

`ppl_new_MIP_Problem(+Constraint_System, +Lin_Expr, +Optimization_Mode, -Handle)`

*Creates an MIP Problem MIP with the feasible region represented by `Constraint_System`, objective function `Lin_Expr` and optimization mode `Optimization_Mode`. `Handle` is unified with the handle for MIP.*

`ppl_new_MIP_Problem_from_MIP_Problem(+Handle_1, -Handle_2)`

*Creates an MIP Problem MIP from the MIP Problem referenced by `Handle_1`. `Handle_2` is unified with the handle for MIP.*

`ppl_MIP_Problem_swap(+Handle_1, +Handle_2)`

*Swaps the MIP Problem referenced by `Handle_1` with the one referenced by `Handle_2`.*

`ppl_delete_MIP_Problem(+Handle)`

*Deletes the MIP Problem referenced by `Handle`. After execution, `Handle` is no longer a valid handle for a PPL MIP Problem.*

`ppl_MIP_Problem_space_dimension(+Handle, ?Dimension_Type)`

*Unifies the dimension of the vector space in which the MIP Problem referenced by `Handle` is embedded with `Dimension_Type`.*

`ppl_MIP_Problem_integer_space_dimensions(+Handle, ?Vars_List)`

*Unifies `Vars_List` with a list of variables representing the integer space dimensions of the MIP Problem referenced by `Handle`.*

`ppl_MIP_Problem_constraints(+Handle, -Constraint_System)`

*Unifies `Constraint_System` with a list of the constraints in the constraints system representing the feasible region for the MIP Problem referenced by `Handle`.*

`ppl_MIP_Problem_objective_function(+Handle, ?Lin_Expr)`

*Unifies `Lin_Expr` with the objective function for the MIP Problem referenced by `Handle`.*



```
ppl_MIP_Problem_optimization_mode(+Handle, ?Optimization_Mode)
```

*Unifies Optimization\_Mode with the optimization mode for the MIP Problem referenced by Handle.*

```
ppl_MIP_Problem_clear(+Handle)
```

*Resets the MIP problem referenced by Handle to be the trivial problem with the feasible region the 0-dimensional universe, objective function 0 and optimization mode max.*

```
ppl_MIP_Problem_add_space_dimensions_and_embed( +Handle, +Dimension_-
Type)
```

*Embeds the MIP problem referenced by Handle in a space that is enlarged by Dimension\_Type dimensions.*

```
ppl_MIP_Problem_add_to_integer_space_dimensions(+Handle, +Vars_List)
```

*Updates the MIP Problem referenced by Handle so that the variables in Vars\_List are added to the set of integer space dimensions.*

```
ppl_MIP_Problem_add_constraint(+Handle, +Constraint)
```

*Updates the MIP Problem referenced by Handle so that the feasible region is represented by the original constraint system together with the constraint Constraint.*

```
ppl_MIP_Problem_add_constraints(+Handle, +Constraint_System)
```

*Updates the MIP Problem referenced by Handle so that the feasible region is represented by the original constraint system together with all the constraints in Constraint\_System.*

```
ppl_MIP_Problem_set_objective_function(+Handle, +Lin_Expr)
```

*Updates the MIP Problem referenced by Handle so that the objective function is changed to Lin\_Expr.*

```
ppl_MIP_Problem_set_control_parameter(+Handle, +Control_Parameter_-
Value)
```

*Updates the MIP Problem referenced by Handle so that the value for the relevant control parameter name is changed to Control\_Parameter\_Value.*

```
ppl_MIP_Problem_get_control_parameter(+Handle, +Control_Parameter_-
Name, ?Control_Parameter_Value)
```

*Unifies Control\_Parameter\_Value with the value of the control parameter Control\_Parameter\_Name.*

```
ppl_MIP_Problem_set_optimization_mode(+Handle, +Optimization_Mode)
```

*Updates the MIP Problem referenced by Handle so that the optimization mode is changed to Optimization\_Mode.*

```
ppl_MIP_Problem_is_satisfiable(+Handle)
```

*Succeeds if and only if the MIP Problem referenced by Handle is satisfiable.*

```
ppl_MIP_Problem_solve(+Handle, ?MIP_Problem_Status)
```

*Solves the MIP problem referenced by Handle and unifies MIP\_Problem\_Status with: unfeasible, if the MIP problem is not satisfiable; unbounded, if the MIP problem is satisfiable but there is no finite bound to the value of the objective function; optimized, if the MIP problem admits an optimal solution.*

```
ppl_MIP_Problem_feasible_point(+Handle, ?Generator)
```

*Unifies Generator with a feasible point for the MIP problem referenced by Handle.*

```
ppl_MIP_Problem_optimizing_point(+Handle, ?Generator)
```

*Unifies Generator with an optimizing point for the MIP problem referenced by Handle.*

```
ppl_MIP_Problem_optimal_value(+Handle, ?Coeff_1, ?Coeff_2)
```

*Unifies Coeff\_1 and Coeff\_2 with the numerator and denominator, respectively, for the optimal value for the MIP problem referenced by Handle.*

```
ppl_MIP_Problem_evaluate_objective_function(+Handle, +Generator,
?Coeff_1, ?Coeff_2)
```

*Evaluates the objective function of the MIP problem referenced by Handle at point Generator. Coefficient\_1 is unified with the numerator and Coefficient\_2 is unified with the denominator of the objective function value at Generator.*

```
ppl_MIP_Problem_OK(+Handle)
```

*Succeeds only if the MIP Problem referenced by Handle is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

## Predicates for C Polyhedra

Here we provide a short description for each of the predicates available for the domain of C polyhedra. Note that predicates for other domains will follow a similar pattern.

### Constructor, copy, conversion and destructor predicates

**Constructor predicates for C polyhedra** The constructor predicates build a C polyhedron from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs. For example, a C polyhedron can be built from a list of non-strict inequality or equality constraints or a list of equality congruences or a list of generators that contains no closure points.

```
ppl_new_C_Polyhedron_from_space_dimension(+Dimension_Type, +Universe_
or_Empty, -Handle)
```

*Builds a C polyhedron  $\mathcal{P}$  with Dimension\_Type dimensions; it is empty or the universe depending on whether Atom is empty or universe, respectively. Handle is unified with the handle for  $\mathcal{P}$ . Thus the query*

```
?- ppl_new_C_Polyhedron_from_space_dimension(3, universe, X).
```

*creates the C polyhedron defining the 3-dimensional vector space  $\mathbb{R}^3$  with X bound to a valid handle for accessing it.*

```
ppl_new_C_Polyhedron_from_constraints(+Constraint_System, -Handle)
```

*Builds a new C polyhedron P from Constraint\_System. Handle is unified with the handle for P.*

```
ppl_new_C_Polyhedron_from_congruences(+Congruence_System, -Handle)
```

*Builds a new C polyhedron P from Congruence\_System. Handle is unified with the handle for P.*

```
ppl_new_C_Polyhedron_from_generators(+Generator_System, -Handle)
```

*Builds a new C polyhedron P from Generator\_System. Handle is unified with the handle for P.*

**Predicates that build new C polyhedra by copying or converting from other semantic geometric descriptions** Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class (e.g., `ppl_new_Grid_from_C_Polyhedron`, `ppl_new_C_Polyhedron_from_BD_Shape_mpq_class`, etc.).

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_C_Polyhedron_from_C_Polyhedron(+Handle_1, -Handle_2)
```

*Builds a new C polyhedron P\_1 from a C polyhedron referenced by handle Handle\_1. Handle\_2 is unified with the handle for P\_1.*

```
ppl_new_C_Polyhedron_from_NNC_Polyhedron(+Handle_1, -Handle_2)
```

*Builds a new C polyhedron P\_1 from an NNC polyhedron referenced by handle Handle\_1. Handle\_2 is unified with the handle for P\_1.*

```
ppl_new_C_polyhedron_from_C_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

*Builds a new C polyhedron P\_1 from a C polyhedron referenced by handle Handle\_1 using an algorithm whose complexity does not exceed Complexity; Handle\_2 is unified with the handle for P\_1.*

```
ppl_new_C_polyhedron_from_NNC_Polyhedron_with_complexity(+Handle,
+Complexity, -Handle)
```

*Builds a new C polyhedron P\_1 from an NNC polyhedron referenced by handle Handle\_1 using an algorithm whose complexity does not exceed Complexity; Handle\_2 is unified with the handle for P\_1.*

**Destructor predicate** Below is the destructor predicate for the Polyhedron domain.

```
ppl_delete_Polyhedron(+Handle)
```

*Invalidates the handle referenced by Handle: this makes sure the corresponding resources will eventually be released.*

### Predicates that do not change the polyhedron

**Test Predicates** These predicates test the polyhedron for different properties and succeed or fail depending on the outcome.

```
ppl_Polyhedron_is_empty(+Handle)
```

*Succeeds if and only if the polyhedron referenced by Handle is empty.*

```
ppl_Polyhedron_is_universe(+Handle)
```

*Succeeds if and only if the polyhedron referenced by Handle is the universe.*

```
ppl_Polyhedron_is_bounded(+Handle)
```

*Succeeds if and only if the polyhedron referenced by Handle is bounded.*

```
ppl_Polyhedron_contains_integer_point(+Handle)
```

*Succeeds if and only if the polyhedron referenced by Handle contains an integer point.*

`ppl_Polyhedron_is_topologically_closed(+Handle)`

*Succeeds if and only if the polyhedron referenced by Handle is topologically closed.*

`ppl_Polyhedron_is_discrete(+Handle)`

*Succeeds if and only if the polyhedron referenced by Handle is discrete.*

`ppl_Polyhedron_bounds_from_above(+Handle, +Lin_Expr)`

*Succeeds if and only if Lin\_Expr is bounded from above in the polyhedron referenced by Handle.*

`ppl_Polyhedron_bounds_from_below(+Handle, +Lin_Expr)`

*Succeeds if and only if Lin\_Expr is bounded from below in the polyhedron referenced by Handle.*

`ppl_Polyhedron_contains_Polyhedron(+Handle_1, +Handle_2)`

*Succeeds if and only if the polyhedron referenced by Handle\_2 is included in or equal to the polyhedron referenced by Handle\_1.*

`ppl_Polyhedron_strictly_contains_Polyhedron(+Handle_1, +Handle_2)`

*Succeeds if and only if the polyhedron referenced by Handle\_2 is included in but not equal to the polyhedron referenced by Handle\_1.*

`ppl_Polyhedron_is_disjoint_from_Polyhedron(+Handle_1, +Handle_2)`

*Succeeds if and only if the polyhedron referenced by Handle\_2 is disjoint from the polyhedron referenced by Handle\_1.*

`ppl_Polyhedron_equals_Polyhedron(+Handle_1, +Handle_2)`

*Succeeds if and only if the polyhedron referenced by Handle\_1 is equal to the polyhedron referenced by Handle\_2.*

`ppl_Polyhedron_OK(+Handle)`

*Succeeds only if the polyhedron referenced by Handle is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

`ppl_Polyhedron_constrains(+Handle, +PPL_Var)`

*Succeeds if and only if the polyhedron referenced by Handle constrains the dimension PPL\_Var.*

**Predicates that return information about the polyhedron** These predicates will obtain more detailed information about the polyhedron unifying some of their arguments with the results.

`ppl_Polyhedron_space_dimension(+Handle, ?Dimension_Type)`

*Unifies Dimension\_Type with the dimension of the vector space enclosing the polyhedron referenced by Handle.*

`ppl_Polyhedron_affine_dimension(+Handle, ?Dimension_Type)`

*Unifies Dimension\_Type with the affine dimension of the polyhedron referenced by Handle.*

`ppl_Polyhedron_relation_with_constraint(+Handle, +Constraint, ?Relation_List)`

*Unifies Relation\_List with the list of relations the polyhedron referenced by Handle has with Constraint. The possible relations are listed in the grammar rules above.*

`ppl_Polyhedron_relation_with_generator(+Handle, +Generator, ?Relation_List)`

*Unifies Relation\_List with the list of relations the polyhedron referenced by Handle has with Generator. The possible relations are listed in the grammar rules above.*

```
ppl_Polyhedron_relation_with_congruence(+Handle, +Congruence,
?Relation_List)
```

*Unifies Relation\_List with the list of relations the polyhedron referenced by Handle has with Congruence. The possible relations are listed in the grammar rules above.*

```
ppl_Polyhedron_get_constraints(+Handle, ?Constraint_System)
```

*Unifies Constraint\_System with the constraints (in the form of a list) in the constraint system satisfied by the polyhedron referenced by Handle.*

```
ppl_Polyhedron_get_generators(+Handle, ?Generator_System)
```

*Unifies Generator\_System with the generators (in the form of a list) in the generator system for the polyhedron referenced by Handle.*

```
ppl_Polyhedron_get_congruences(+Handle, ?Congruence_System)
```

*Unifies Congruence\_System with the congruences (in the form of a list) in the congruence system satisfied by the polyhedron referenced by Handle.*

```
ppl_Polyhedron_get_minimized_constraints(+Handle, ?Constraint_System)
```

*Unifies Constraint\_System with the constraints (in the form of a list) in the minimized constraint system satisfied by the polyhedron referenced by Handle.*

```
ppl_Polyhedron_get_minimized_generators(+Handle, ?Generator_System)
```

*Unifies Generator\_System with the generators (in the form of a list) in the minimized generator system for the polyhedron referenced by Handle.*

```
ppl_Polyhedron_get_minimized_congruences(+Handle, ?Congruence_System)
```

*Unifies Congruence\_System with the congruences (in the form of a list) in the minimized congruence system satisfied by the polyhedron referenced by Handle.*

```
ppl_Polyhedron_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

*Succeeds if and only if polyhedron P referenced by Handle is not empty and Lin\_Expr is bounded from above in P.*

*Coeff\_1 is unified with the numerator of the supremum value and Coeff\_2 with the denominator of the supremum value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.*

```
ppl_Polyhedron_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2,
?Boolean)
```

*Succeeds if and only if polyhedron P referenced by Handle is not empty and Lin\_Expr is bounded from below in P.*

*Coeff\_1 is unified with the numerator of the infimum value and Coeff\_2 with the denominator of the infimum value. If the infimum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.*

```
ppl_Polyhedron_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

*Succeeds if and only if polyhedron P referenced by Handle is not empty and Lin\_Expr is bounded from above in P.*

*Coeff\_1 is unified with the numerator of the supremum value and Coeff\_2 with the denominator of the supremum value and Point with a point or closure point where Lin\_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.*

```
ppl_Polyhedron_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1,
?Coeff_2, ?Boolean, ?Point)
```

*Succeeds if and only if polyhedron P referenced by Handle is not empty and Lin\_Expr is bounded from below in P.*

*Coeff\_1 is unified with the numerator of the infimum value and Coeff\_2 with the denominator of the infimum value and Point with a point or closure point where Lin\_Expr reaches this value. If the infimum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.*

```
ppl_Polyhedron_external_memory_in_bytes(+Handle, ?Number)
```

*Unifies Number with the size of the total memory in bytes occupied by the polyhedron referenced by Handle.*

```
ppl_Polyhedron_total_memory_in_bytes(+Handle, ?Number)
```

*Unifies Number with the size of the external memory in bytes occupied by the polyhedron referenced by Handle.*

### Space-dimension preserving predicates that may change the polyhedron

These predicates may modify the polyhedron referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

**Predicates that may change the polyhedron by adding to the polyhedron's constraint or generator descriptions** Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section "Generic Operations on Semantic Geometric Descriptors" in the main PPL user manual for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Polyhedron_add_constraint(+Handle, +Constraint)
```

*Updates the polyhedron referenced by Handle to one obtained by adding Constraint to its constraint system. For a C polyhedron, Constraint must be an equality or a non-strict inequality.*

```
ppl_Polyhedron_add_congruence(+Handle, +Congruence)
```

*Updates the polyhedron referenced by Handle to one obtained by adding Congruence to its congruence system. For a C polyhedron, Congruence must be an equality.*

```
ppl_Polyhedron_add_generator(+Handle, +Generator)
```

*Updates the polyhedron referenced by Handle to one obtained by adding Generator to its generator system. For a C polyhedron, Generator must be a line, ray or point.*

```
ppl_Polyhedron_add_constraints(+Handle, +Constraint_System)
```

*Updates the polyhedron referenced by Handle to one obtained by adding to its constraint system the constraints in Constraint\_System. For a C polyhedron, Constraints must be a list of equalities and non-strict inequalities.*

```
ppl_Polyhedron_add_congruences(+Handle, +Congruence_System)
```

*Updates the polyhedron referenced by Handle to one obtained by adding to its congruence system the congruences in Congruence\_System. For a C polyhedron, Congruences must be a list of equalities.*

```
ppl_Polyhedron_add_generators( +Handle, +Generator_System)
```

*Updates the polyhedron referenced by Handle to one obtained by adding to its generator system the generators in Generator\_System. For a C polyhedron, Generators must be a list of lines, rays and points.*

```
ppl_Polyhedron_refine_with_constraint( +Handle, +Constraint)
```

*Updates the polyhedron referenced by Handle to one obtained by refining its constraint system with Constraint.*

```
ppl_Polyhedron_refine_with_congruence( +Handle, +Congruence)
```

*Updates the polyhedron referenced by Handle to one obtained by refining its congruence system with Congruence.*

```
ppl_Polyhedron_refine_with_constraints( +Handle, +Constraint_System)
```

*Updates the polyhedron referenced by Handle to one obtained by refining its constraint system with the constraints in Constraint\_System.*

```
ppl_Polyhedron_refine_with_congruences( +Handle, +Congruence_System)
```

*Updates the polyhedron referenced by Handle to one obtained by refining its congruence system with the congruences in Congruence\_System.*

**Predicates that transform a polyhedron** These predicates enable transformations such as taking the topological closure (which for the domain of C polyhedron is the identity transformation), unconstraining a specified dimension as explained in the main PPL user manual in Section "Cylindrification Operator" and several different image and preimage affine transfer relations; for details of the latter see Sections "Images and Preimages of Affine Transfer Relations" and "Generalized Affine Relations".

```
ppl_Polyhedron_topological_closure_assign(+Handle)
```

*Assigns to the polyhedron referenced by Handle its topological closure.*

```
ppl_Polyhedron_unconstrain_space_dimension(+Handle, +PPL_Var)
```

*Modifies the polyhedron P referenced by Handle by unconstraining the space dimension PPL\_Var.*

```
ppl_Polyhedron_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)
```

*Modifies the polyhedron P referenced by Handle by unconstraining the space dimensions that are specified in List\_of\_PPL\_Var. The presence of duplicates in List\_of\_PPL\_Var is a waste but an innocuous one.*

```
ppl_Polyhedron_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

*Transforms the polyhedron referenced by Handle assigning the affine expression for Lin\_Expr/Coeff to PPL\_Var.*

```
ppl_Polyhedron_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

*Transforms the polyhedron referenced by Handle substituting the affine expression for Lin\_Expr/Coeff to PPL\_Var.*

```
ppl_Polyhedron_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

*Assigns to polyhedron P referenced by Handle the generalized image with respect to the generalized affine transfer relation  $\text{Lin\_Expr}_1/\text{Coeff} \leq \text{PPL\_Var} \leq \text{Lin\_Expr}_2/\text{Coeff}$ .*



```
ppl_Polyhedron_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1,
+Lin_Expr_2, +Coeff)
```

*Assigns to polyhedron P referenced by Handle the generalized preimage with respect to the generalized affine transfer relation  $\text{Lin\_Expr}_1/\text{Coeff} \leq \text{PPL\_Var} \leq \text{Lin\_Expr}_2/\text{Coeff}$ .*

```
ppl_Polyhedron_generalized_affine_image(+Handle, +PPL_Var, +Relation_
Symbol, +Lin_Expr, +Coeff)
```

*Assigns to polyhedron P referenced by Handle the generalized image with respect to the generalized affine transfer relation  $\text{PPL\_Var} \bowtie \text{Lin\_Expr}/\text{Coeff}$ , where  $\bowtie$  is the symbol represented by Relation\_Symbol.*

```
ppl_Polyhedron_generalized_affine_preimage(+Handle, +PPL_Var,
+Relation_Symbol, +Lin_Expr, +Coeff)
```

*Assigns to polyhedron P referenced by Handle the generalized preimage with respect to the generalized affine transfer relation  $\text{PPL\_Var} \bowtie \text{Lin\_Expr}/\text{Coeff}$ , where  $\bowtie$  is the symbol represented by Relation\_Symbol.*

```
ppl_Polyhedron_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1,
+Relation_Symbol, +Lin_Expr_2)
```

*Assigns to polyhedron P referenced by Handle the generalized image with respect to the generalized affine transfer relation  $\text{Lin\_Expr}_1 \bowtie \text{Lin\_Expr}_2$ , where  $\bowtie$  is the symbol represented by Relation\_Symbol.*

```
ppl_Polyhedron_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_
Expr_1, +Relation_Symbol, +Lin_Expr_2)
```

*Assigns to polyhedron P referenced by Handle the generalized preimage with respect to the generalized affine transfer relation  $\text{Lin\_Expr}_1 \bowtie \text{Lin\_Expr}_2$ , where  $\bowtie$  is the symbol represented by Relation\_Symbol.*

### Predicates whose results depend on more than one polyhedron

These predicates include the binary operators which will assign to the polyhedron referred to by the first argument its combination with the polyhedron referred to by the second argument as described in the main PPL user manual in Sections "Intersection and Convex Polyhedral Hull" and "Convex Polyhedral Difference"; and a linear partitioning operator described below.

```
ppl_Polyhedron_intersection_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron P referenced by Handle\_1 the intersection of P and the polyhedron referenced by Handle\_2.*

```
ppl_Polyhedron_upper_bound_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron P referenced by Handle\_1 the upper bound of P and the polyhedron referenced by Handle\_2.*

```
ppl_Polyhedron_difference_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron P referenced by Handle\_1 the difference of P and the polyhedron referenced by Handle\_2.*

```
ppl_Polyhedron_time_elapse_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron P referenced by Handle\_1 the time elapse of P and the polyhedron referenced by Handle\_2.*

```
ppl_Polyhedron_poly_hull(+Handle_1, +Handle_2)
```



*Assigns to the polyhedron  $P$  referenced by `Handle_1` the poly-hull of  $P$  and the polyhedron referenced by `Handle_2`.*

```
ppl_Polyhedron_poly_difference(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by `Handle_1` the poly-difference of  $P$  and the polyhedron referenced by `Handle_2`.*

```
ppl_Polyhedron_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

*Succeeds if the least upper bound of the polyhedron  $P_1$  referenced by `Handle_1` with the polyhedron referenced by `Handle_2` is exact; in which case the least upper bound is assigned to  $P_1$ ; fails otherwise.*

```
ppl_Polyhedron_poly_hull_assign_if_exact(+Handle_1, +Handle_2)
```

*Succeeds if the least upper bound of the polyhedron  $P_1$  referenced by `Handle_1` with the polyhedron referenced by `Handle_2` is exact; in which case the least upper bound is assigned to  $P_1$ ; fails otherwise.*

```
ppl_Polyhedron_simplify_using_context_assign(+Handle_1, +Handle_2,  
?Boolean)
```

*Succeeds if and only if the intersection of polyhedron  $P_1$  referenced by `Handle_1` and the polyhedron  $P_2$  referenced by `Handle_2` is non-empty. Assigns to  $P_1$  its meet-preserving simplification with respect to  $P_2$ .*

```
ppl_Polyhedron_linear_partition(+Handle_1, +Handle_2, -Handle_3,  
-Handle_4)
```

*`Handle_1` and `Handle_2` are handles for elements  $P_1$  and  $P_2$  in the Polyhedron domain. The predicate unifies handle `Handle_3` to a reference to the intersection of  $P_1$  and  $P_2$  and `Handle_4` to a reference to a pointset powerset of nnc polyhedra  $P_4$ ; where  $P_4$  is the linear partition of  $P_1$  with respect to  $P_2$ . This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.*

**Predicates for widening and extrapolation** In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main PPL user manual in Sections "Widening Operators", "Widening with Tokens" and "Extrapolation Operators". Note that for all these widening and extrapolation predicates to behave as specified the polyhedron referred to by the second argument has to be contained in (or equal to) the polyhedron referred to by the first argument.

```
ppl_Polyhedron_BHRZ03_widening_assign_with_tokens(+Handle_1, +Handle_2,  
+C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron  $P_1$  referenced by `Handle_1` the BHRZ03-widening of  $P_1$  with the polyhedron referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_H79_widening_assign_with_tokens(+Handle_1, +Handle_2,  
+C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron  $P_1$  referenced by `Handle_1` the H79-widening of  $P_1$  with the polyhedron referenced by `Handle_2`. The widening with tokens delay technique is applied with `C_unsigned_1` tokens; `C_unsigned_2` is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_BHRZ03_widening_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P_1$  referenced by `Handle_1` the BHRZ03-widening of  $P_1$  with the polyhedron referenced by `Handle_2`.*

```
ppl_Polyhedron_H79_widening_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron P\_1 referenced by Handle\_1 the H79-widening of P\_1 with the polyhedron referenced by Handle\_2.*

```
ppl_Polyhedron_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_
unsigned_1, ?C_unsigned_2)
```

*Same as the predicate ppl\_Polyhedron\_H79\_widening\_assign\_with\_tokens/4*

```
ppl_Polyhedron_widening_assign(+Handle_1, +Handle_2)
```

*Same as the predicate ppl\_Polyhedron\_H79\_widening\_assign/2*

```
ppl_Polyhedron_limited_BHRZ03_extrapolation_assign_with_
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_
unsigned_2)
```

*Assigns to the polyhedron P\_1 referenced by Handle\_1 the BHRZ03-widening of P\_1 with the polyhedron referenced by Handle\_2 intersected with the constraints in Constraint\_System that are satisfied by all the points of P\_1. The widening with tokens delay technique is applied with C\_unsigned\_1 tokens; C\_unsigned\_2 is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_limited_H79_extrapolation_assign_with_tokens(+Handle_1,
+Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron P\_1 referenced by Handle\_1 the H79-widening of P\_1 with the polyhedron referenced by Handle\_2 intersected with the constraints in Constraint\_System that are satisfied by all the points of P\_1. The widening with tokens delay technique is applied with C\_unsigned\_1 tokens; C\_unsigned\_2 is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_bounded_BHRZ03_extrapolation_assign_with_
tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_
unsigned_2)
```

*Assigns to the polyhedron P\_1 referenced by Handle\_1 the BHRZ03-widening of P\_1 with the polyhedron referenced by Handle\_2 intersected with the constraints in Constraint\_System that are satisfied by all the points of P\_1, further intersected with the smallest box containing p P\_1.. The widening with tokens delay technique is applied with C\_unsigned\_1 tokens; C\_unsigned\_2 is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_bounded_H79_extrapolation_assign_with_tokens(+Handle_1,
+Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron P\_1 referenced by Handle\_1 the H79-widening of P\_1 with the polyhedron referenced by Handle\_2 intersected with the constraints in Constraint\_System that are satisfied by all the points of P\_1, further intersected with the smallest box containing p P\_1.. The widening with tokens delay technique is applied with C\_unsigned\_1 tokens; C\_unsigned\_2 is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_limited_BHRZ03_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

*Assigns to the polyhedron P\_1 referenced by Handle\_1 the BHRZ03-widening of P\_1 with the polyhedron referenced by Handle\_2 intersected with the constraints in Constraint\_System that are satisfied by all the points of P\_1.*

```
ppl_Polyhedron_limited_H79_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

*Assigns to the polyhedron P\_1 referenced by Handle\_1 the H79-widening of P\_1 with the polyhedron referenced by Handle\_2 intersected with the constraints in Constraint\_System that are satisfied by all the points of P\_1.*

```
ppl_Polyhedron_bounded_BHRZ03_extrapolation_assign(+Handle_1,
+Handle_2, +Constraint_System)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the BHRZ03-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ , further intersected with the smallest box containing  $P_1$ .*

```
ppl_Polyhedron_bounded_H79_extrapolation_assign(+Handle_1, +Handle_2,
+Constraint_System)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the H79-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ , further intersected with the smallest box containing  $P_1$ .*

### Predicates that may modify the vector space of the polyhedron

These predicates enable the modification of the vector space of the polyhedron referred to in the first argument. Detailed descriptions of these can be found in the main PPL user manual in Sections "Concatenating Polyhedra", "Adding New Dimensions to the Vector Space", "Removing Dimensions from the Vector Space", "Mapping the Dimensions of the Vector Space", "Expanding One Dimension of the Vector Space to Multiple Dimensions" and "Folding Multiple Dimensions of the Vector Space into One Dimension".

```
ppl_Polyhedron_concatenate_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by  $Handle_1$  the concatenation of  $P$  and the polyhedron referenced by  $Handle_2$ .*

```
ppl_Polyhedron_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

*Adds  $Dimension\_Type$  new dimensions to the space enclosing the polyhedron  $P$  referenced by  $Handle$  and embeds  $P$  in this space.*

```
ppl_Polyhedron_add_space_dimensions_and_project(+Handle, +Dimension_Type)
```

*Adds  $Dimension\_Type$  new dimensions to the space enclosing the polyhedron  $P$  referenced by  $Handle$  and projects  $P$  in this space.*

```
ppl_Polyhedron_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

*Removes from the vector space enclosing the polyhedron  $P$  referenced by  $Handle$  the space dimensions that are specified in  $List\_of\_PPL\_Var$ . The presence of duplicates in  $List\_of\_PPL\_Var$  is a waste but an innocuous one.*

```
ppl_Polyhedron_remove_higher_space_dimensions(+Handle, +Dimension_Type)
```

*Removes the higher dimensions from the vector space enclosing the polyhedron  $P$  referenced by  $Handle$  so that, upon successful return, the new space dimension is  $Dimension\_Type$ .*

```
ppl_Polyhedron_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)
```

*Expands the  $PPL\_Var$ -th dimension of the vector space enclosing the polyhedron referenced by  $Handle$  to  $Dimension\_Type$  new space dimensions.*

```
ppl_Polyhedron_fold_space_dimensions(+Handle, +List_of_PPL_Vars,
+PPL_Var)
```

*Modifies the polyhedron referenced by  $Handle$  by folding the space dimensions contained in  $List\_of\_PPL\_Vars$  into dimension  $PPL\_Var$ . The presence of duplicates in  $List\_of\_PPL\_Vars$  is a waste*

*but an innocuous one.*

```
ppl_Polyhedron_map_space_dimensions(+Handle, +P_Func)
```

*Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the P\_Func, which has n entries. The result is undefined if P\_Func does not encode a partial function.*

### Ascii dump predicate

```
ppl_Polyhedron_ascii_dump(+Handle)
```

*Dumps an ascii representation of the PPL internal state for polyhedron referenced by Handle on the standard output. Useful for debugging.*

## Ad hoc Predicates for Other Domains

### Extra Predicates Specifically for the Pointset\_Powerset Domains

The powerset domains can be instantiated by taking as a base domain any fixed semantic geometric description (C and NNC polyhedra, BD and octagonal shapes, boxes and grids). An element of the powerset domain represents a disjunctive collection of base objects (its disjuncts), all having the same space dimension.

Besides the predicates that are available in all semantic geometric descriptions (whose documentation is not repeated here), the powerset domain also provides several ad hoc functions. In particular, the iterator types allow for the examination and manipulation of the collection of disjuncts.

Below is a specification of extra predicates provided for the Pointset\_Powerset<C\_Polyhedron> domain. Note that predicates for other Pointset\_Powerset domains will follow a similar pattern.

### Predicates for pointset powerset iterators.

```
ppl_new_Pointset_Powerset_C_Polyhedron_iterator_from_iterator_pps_-
iter_doc(+Iterator_1, -Iterator_2)
```

*Builds a new iterator it from the iterator referenced by Iterator\_1. Iterator\_2 is unified with the handle for it.*

```
ppl_Pointset_Powerset_C_Polyhedron_begin_iterator(+Handle, -Iterator)
```

*Unifies Iterator with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by Handle.*

```
ppl_Pointset_Powerset_C_Polyhedron_end_iterator(+Handle, -Iterator)
```

*Unifies Iterator with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by Handle.*

```
ppl_Pointset_Powerset_C_Polyhedron_iterator_equals_-
iterator(+Iterator_1, +Iterator_2)
```

*Succeeds if and only if the iterator referenced by Iterator\_1 is equal to the iterator referenced by Iterator\_2.*

```
ppl_Pointset_Powerset_C_Polyhedron_iterator_increment(+Iterator)
```

*Increments the iterator referenced by Iterator so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_C_Polyhedron_iterator_decrement(+Iterator)
```

*Decrements the iterator referenced by Iterator so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_C_Polyhedron_iterator_get_disjunct(+Iterator,  
-Handle)
```

*Unifies with Handle a reference to the disjunct referred to by Iterator\_1.*

```
ppl_delete_Pointset_Powerset_C_Polyhedron_iterator(+Iterator)
```

*Invalidates the handle referenced by Iterator: this makes sure the corresponding resources will eventually be released.*

### Other Ad Hoc Predicates for the Pointset Powerset.

```
ppl_Pointset_Powerset_C_Polyhedron_pairwise_reduce(+Handle)
```

*Assigns the result of pairwise reduction on the pointset powerset referenced by Handle.*

```
ppl_Pointset_Powerset_C_Polyhedron_omega_reduce(+Handle)
```

*Assigns the result of omega reduction on the pointset powerset referenced by Handle.*

```
ppl_Pointset_Powerset_C_Polyhedron_geometrically_covers_Pointset_  
Powerset_C_Polyhedron(+Handle_1, +Handle_2)
```

*Succeeds if and only if the pointset powerset referenced by Handle\_2 geometrically covers the pointset powerset referenced by Handle\_1.*

```
ppl_Pointset_Powerset_C_Polyhedron_geometrically_equals_Pointset_  
Powerset_C_Polyhedron(+Handle_1, +Handle_2)
```

*Succeeds if and only if the pointset powerset referenced by Handle\_2 geometrically equals the pointset powerset referenced by Handle\_1.*

```
ppl_Pointset_Powerset_C_Polyhedron_add_disjunct(+Handle_1, +Handle_2)
```

*Adds to the pointset powerset referenced by Handle\_1 a disjunct referred to by Handle\_2.*

```
ppl_Pointset_Powerset_C_Polyhedron_drop_disjunct(+Handle, +Iterator)
```

*If it is the iterator referred to by Iterator, drops from the pointset powerset referenced by Handle the disjunct pointed to by it and assigns to it an iterator to the next disjunct.*

```
ppl_Pointset_Powerset_C_Polyhedron_drop_disjuncts(+Handle, +Iterator_  
1, +Iterator_2)
```

*If it\_1 and it\_2 are the iterators referred to by Iterator\_1 and Iterator\_2, respectively, drops from the pointset powerset referenced by Handle all the disjuncts from it\_1 to it\_2 (excluded).*

```
ppl_Pointset_Powerset_C_Polyhedron_BHZ03_BHRZ03_BHRZ03_widening_  
assign(+Handle_1, +Handle_2)
```

*Assigns to the pointset powerset P\_1 referenced by Handle\_1 the BHZ03-widening between P\_1 and the pointset powerset referenced by Handle\_2, using the BHRZ03-widening certified by the convergence certificate for BHRZ03.*

```
ppl_Pointset_Powerset_C_Polyhedron_BHZ03_H79_H79_widening_  
assign(+Handle_1, +Handle_2)
```

*Assigns to the pointset powerset P\_1 referenced by Handle\_1 the BHZ03-widening between P\_1 and the pointset powerset referenced by Handle\_2, using the H79-widening certified by the convergence certificate for H79.*

```
ppl_Pointset_Powerset_C_Polyhedron_BGP99_BHRZ03_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

*Assigns to the pointset powerset P\_1 referenced by Handle\_1 the result of applying the BGP99 extrapolation operator between P\_1 and the pointset powerset referenced by Handle\_2, using the BHRZ03-widening and the cardinality threshold C\_unsigned.*

```
ppl_Pointset_Powerset_C_Polyhedron_BGP99_H79_extrapolation_-
assign(+Handle_1, +Handle_2, C_unsigned)
```

*Assigns to the pointset powerset P\_1 referenced by Handle\_1 the result of applying the BGP99 extrapolation operator between P\_1 and the pointset powerset referenced by Handle\_2, using the H79-widening and the cardinality threshold C\_unsigned.*

## Compilation and Installation

When the Parma Polyhedra Library is configured, it tests for the existence of each supported Prolog system. If a supported Prolog system is correctly installed in a standard location, things are arranged so that the corresponding interface is built and installed.

As an option, the Prolog interface can track the creation and disposal of polyhedra. In fact, differently from native Prolog data, PPL polyhedra must be explicitly disposed and forgetting to do so is a very common mistake. To enable this option, configure the library adding `-DPROLOG_TRACK_ALLOCATION` to the options passed to the C++ compiler. Your configure command would then look like

```
path/to/configure --with-cxxflags="-DPROLOG_TRACK_ALLOCATION" ...
```

## System-Dependent Features

### CIAO Prolog

The Ciao Prolog interface to the PPL is available both as ‘PPL enhanced’ Ciao Prolog interpreter and as a library that can be linked to Ciao Prolog programs. Only Ciao Prolog versions 1.10 #5 and later are supported.

So that it can be used with the Ciao Prolog PPL interface, the Ciao Prolog installation must be configured with the `-disable-regs` option.

**The `ppl_ciao` Executable** If an appropriate version of Ciao Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_ciao` in the directory `prefix/bin`. The `ppl_ciao` executable is simply the Ciao Prolog interpreter with the Parma Polyhedra Library linked in. The only thing you should do to use the library is to call `ppl_initialize/0` before any other PPL predicate and to call `ppl_finalize/0` when you are done with the library.

**Linking the Library To Ciao Prolog Programs** In order to allow linking Ciao Prolog programs to the PPL, the following files are installed in the directory `prefix/lib/ppl`: `ppl_ciao.pl` contains the required foreign declarations; `libppl_ciao.*` contain the executable code for the Ciao Prolog interface in various formats (static library, shared library, libtool library). If your Ciao Prolog program is constituted by, say, `source1.pl` and `source2.pl` and you want to create the executable `myprog`, your compilation command may look like

```
ciaoc -o myprog prefix/lib/ppl/ppl_ciao.pl ciao_pl_check.pl \
-L '-Lprefix/lib/ppl -lppl_ciao -Lprefix/lib -lppl -lgmpxx -lgmp -lstdc++'
```

## GNU Prolog

The GNU Prolog interface to the PPL is available both as a ‘PPL enhanced’ GNU Prolog interpreter and as a library that can be linked to GNU Prolog programs. The only GNU Prolog version that is known to work is a patched version of the ‘unstable version’ tagged 20040608 (which unpacks to a directory called `gprolog-1.2.18`). The patch is contained in the `interfaces/Prolog/GNU/README` file of the PPL’s distribution.

So that it can be used with the GNU Prolog PPL interface (and, for that matter, with any foreign code), the GNU Prolog installation must be configured with the `-disable-regs` option.

**The `ppl_gprolog` Executable** If an appropriate version of GNU Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_gprolog` in the directory `prefix/bin`. The `ppl_gprolog` executable is simply the GNU Prolog interpreter with the Parma Polyhedra Library linked in. The only thing you should do to use the library is to call `ppl_initialize/0` before any other PPL predicate and to call `ppl_finalize/0` when you are done with the library.

**Linking the Library To GNU Prolog Programs** In order to allow linking GNU Prolog programs to the PPL, the following files are installed in the directory `prefix/lib/ppl`: `ppl_gprolog.pl` contains the required foreign declarations; `libppl_gprolog.*` contain the executable code for the GNU Prolog interface in various formats (static library, shared library, libtool library). If your GNU Prolog program is constituted by, say, `source1.pl` and `source2.pl` and you want to create the executable `myprog`, your compilation command may look like

```
gplc -o myprog prefix/lib/ppl/ppl_gprolog.pl source1.pl source2.pl \
-L '-Lprefix/lib/ppl -lppl_gprolog -Lprefix/lib -lppl -lgmpxx -lgmp -lstdc++'
```

## SICStus Prolog

The SICStus Prolog interface to the PPL is available both as a statically linked module or as a dynamically linked one. Only SICStus Prolog versions 3.9.0 and later are supported.

**The Statically Linked `ppl_sicstus` Executable** If an appropriate version of SICStus Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_sicstus` in the directory `prefix/bin`. The `ppl_sicstus` executable is simply the SICStus Prolog system with the Parma Polyhedra Library statically linked. The only thing you should do to use the library is to load `prefix/lib/ppl/ppl_sicstus.pl`.

**Loading the SICStus Interface Dynamically** In order to dynamically load the library from SICStus Prolog you should simply load `prefix/lib/ppl/ppl_sicstus.pl`. Notice that, for dynamic linking to work, you should have configured the library with the `-enable-shared` option.

## SWI-Prolog

The SWI-Prolog interface to the PPL is available both as a statically linked module or as a dynamically linked one. Only SWI-Prolog version 5.6.0 and later versions are supported.



**The `ppl_pl` Executable** If an appropriate version of SWI-Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_pl` in the directory `prefix/bin`. The `ppl_pl` executable is simply the SWI-Prolog shell with the Parma Polyhedra Library statically linked: from within `ppl_pl` all the services of the library are available without further action.

**Loading the SWI-Prolog Interface Dynamically** In order to dynamically load the library from SWI-Prolog you should simply load `prefix/lib/ppl/ppl_swiprolog.pl`. This will invoke `ppl_initialize/0` and `ppl_finalize/0` automatically. Alternatively, you can load the library directly with

```
:- load_foreign_library('prefix/lib/ppl/libppl_swiprolog').
```

This will call `ppl_initialize/0` automatically. Analogously,

```
:- unload_foreign_library('prefix/lib/ppl/libppl_swiprolog').
```

will, as part of the unload process, invoke `ppl_finalize/0`.

Notice that, for dynamic linking to work, you should have configured the library with the `-enable-shared` option.

## XSB

The XSB Prolog interface to the PPL is available as a dynamically linked module. Only some CVS versions of XSB starting from 2 July 2005 are known to work. CVS versions starting from 11 November 2005 are known not to work.

In order to dynamically load the library from XSB you should load the `ppl_xsb` module and import the predicates you need. For things to work, you may have to copy the files `prefix/lib/ppl/ppl_xsb.xwam` and `prefix/lib/ppl/ppl_xsb.so` in your current directory or in one of the XSB library directories.

## YAP

The YAP Prolog interface to the PPL is available as a dynamically linked module. Only YAP versions following 5.1.0 and CVS HEAD versions starting from 4 January 2006 are supported. Notice that support for unbounded integers in YAP is young and may have errors that could affect programs using the PPL (see, e.g., <http://www.cs.unipr.it/pipermail/ppl-devel/2006-January/007780.html>).

In order to dynamically load the library from YAP you should simply load `prefix/lib/ppl/ppl_yap.pl`. This will invoke `ppl_initialize/0` automatically; it is the programmer's responsibility to call `ppl_finalize/0` when the PPL library is no longer needed. Notice that, for dynamic linking to work, you should have configured the library with the `-enable-shared` option.

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Version 3, 29 June 2007

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