Hybrid Domain Processing

Matt Flax <flatmax@ieee.org>

11th April 2005

1 Introduction

Hybrid domain processing is particularly suited to objective assessment machines. These machines must perform locally correlated assessments on continuous streams of data.

The issue is that an algorithm must asses from causal time zero till infinity. Algorithms such as waveform similarity overlap add (WSOLA) [1] is an example.

Localisation is performed on a potentially infinite stream using a window in the time domain. A window is cheap as it is a simple multiplier. For example over a time period (dt) a Hanning window ($h(t)$) is applied to a data signal ($x(t)$). This is expressed

$$\int h(s-t)x(t)dt \tag{1}$$

and it accounts for localisation at a point in time (s) cheaply.

In the case of objective assessment, many Euclidean operators may perform the actual assessment. Consider for example simple distance assessment ($d(s)$) between a desired match ($y(t)$) and the multiply localised signal in Equation 1. This is expressed as

$$d(s) = h(t)y(t) - \int h(s-t)x(t)dt \tag{2}$$

and describes a limited application over a time interval (s) and a desired signal ($y(t)$). This is a computationally slow assessment. It requires M times N seconds to asses, where $M \wedge N \in \mathcal{R}$, if it is assumed that the convolution is much more computationally complex then multiplication.

2 Hybrid domain processing

If the domain was switched to the Fourier domain, then we may speed the computation by completing a convolution in this different domain. For example let us simplify the convolution to multiplication. In this case we choose the Fourier domain where

$$\mathcal{F}\left\{d(s)\right\} = \mathcal{F}\left\{h(t)y(t) - \int h(s-t)x(t)dt\right\} \tag{3}$$

which may be expressed as

$$\mathcal{F}\left\{d(s)\right\} = \mathcal{F}\left\{h(t)y(t)\right\} - \mathcal{F}\left\{\int h(s-t)x(t)dt\right\} \tag{4}$$

which has the convolution replaced by multiplication in the other domain, i.e.

$$\mathcal{F}\left\{d(s)\right\} = \mathcal{F}\left\{h(t)y(t)\right\} - H(f)X(f) \tag{5}$$

This now adds the computational load of the transform, however removes the computational load of the convolution.

We are left with a computational load of the M seconds plus T seconds, where T is the time lag to transform domains. This is an implementational improvement on Equation 2 which takes MN seconds, as long as

$$2T < (M-1)N \tag{6}$$

this is because it requires a forward and inverse transform ($2T$) and we will have a minimum of M seconds in a constant order estimation system.

3 Conclusion

It is possible to improve implementational efficiency of estimation algorithms which operate in hybrid domains. The efficiency increase is given in Equation 9. If Equation 9 is false then a efficiency decrease is experienced and is expressed as

$$2T > (M-1)N \tag{7}$$

4 Even better speed

It is quicker to compute a different assessment operator, expressed as

$$d(s) = h(t)y(t)(h(-t) * x(t)) \tag{8}$$

where '$*$' is the convolution operator. This equation may be estimated as

$$d(s) = \mathcal{F}\{h(t)y(t)h(-t)\}\mathcal{F}\{x(t)\} \tag{9}$$

References

[1] W. Verhelst and M. Roelands. An overlap-add technique based on waveform similarity (wsola) for high quality time-scale modification of speech. In *IEEE International Conference on Acoustics, Speech, and Signal Processing*, volume 2, pages 554–557, April 1993.